

[09-03-25-T11 REV3]
Essential Facts for trigonometry

[1] $\pi \text{ rad} = 180^\circ$

[2] $s = r \theta$

[3]	$\sin \theta = \frac{y}{r}$	$\csc \theta = \frac{1}{\sin \theta}$
	$\cos \theta = \frac{x}{r}$	$\sec \theta = \frac{1}{\cos \theta}$
	$\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$

[4] $-1 \leq \sin \theta \leq 1$ $-1 \leq \cos \theta \leq 1$

[5] $\sin^2 \theta + \cos^2 \theta = 1$

[6] $\tan^2 \theta + 1 = \sec^2 \theta$

[7] $1 + \cot^2 \theta = \csc^2 \theta$

Consequences of symmetry and periodicity of unit circle

[8] $\sin(\theta + 2n\pi) = \sin \theta$
 $\cos(\theta + 2n\pi) = \cos \theta$
 $\tan(\theta + n\pi) = \tan \theta$

[9] $\sin(-\theta) = -\sin \theta$
 $\cos(-\theta) = \cos(\theta)$
 $\tan(-\theta) = -\tan \theta$

[10]	$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$	$\sin\left(\frac{\pi}{2} + \theta\right) = \cos \theta$
	$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$	$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$
	$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan \theta}$	$\tan\left(\frac{\pi}{2} + \theta\right) = -\frac{1}{\tan \theta}$

[11]	$\sin(\pi - \theta) = \sin \theta$	$\sin(\pi + \theta) = -\sin \theta$
	$\cos(\pi - \theta) = -\cos \theta$	$\cos(\pi + \theta) = -\cos \theta$
	$\tan(\pi - \theta) = -\tan \theta$	$\tan(\pi + \theta) = \tan \theta$

■ Addition formulae

$$[12] \quad \cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$[13] \quad \cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$[14] \quad \sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$[15] \quad \sin(x - y) = \sin x \cos y - \sin y \cos x$$

$$[16] \quad \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$[17] \quad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Consequences of addition formulae

■ Double angle formulae

$$[18] \quad \cos 2x = \cos^2 x - \sin^2 x$$

$$[19] \quad \sin 2x = 2 \sin x \cos x$$

$$[20] \quad \cos 2x = 2 \cos^2 x - 1$$

$$[21] \quad \cos 2x = 1 - 2 \sin^2 x$$

$$[22] \quad \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

■ Half angle formulae

$$[23] \quad \cos^2 x = \frac{1}{2} (1 + \cos 2x)$$

$$[24] \quad \sin^2 x = \frac{1}{2} (1 - \cos 2x)$$

$$[25] \quad \tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

■ Product angle formulae

$$[26] \quad \cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$[27] \quad \sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$[28] \quad \sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$